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14. ABSTRACT
To accomplish feats of rapid, nimble locomotion, the ground mobile robots of tomorrow must exhibit great dexterity and dynamic mobility. As their complexity increases, however, the need for principled approaches to coordinating a robot's actuators becomes apparent. We have begun to pursue a new formalism for solving such problems, making use of Algebraic Topology and

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Dynamic Mobility via Cellular Decompositions of Coordination Spaces: Final Report

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replace traditional methods of combinatorial search and optimization with a computationally easier and conceptually more straightforward approach that identifies and exploits symmetries in a system.

In our approach, we decompose a system into cells that index symmetric configurations. For a multilegged robot negotiating its way through a rubble-strewn terrain, the robot must choose between a number of alternative ways to move its legs. We reduce this problem to changes on a much simpler space of gait timing, using a cell complex to identify all possible gaits, discretizing the space while also providing a roadmap for gait transitions. By studying these representations---and the cellular decompositions that arise---we develop novel approaches to the fundamental control problems necessary for achieving robotic mobility.

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(c) Presentations

Dynamic Bounding with a Passive Compliant Spine, Dynamic Walking Conference, Pensacola, FL, May 2012.
Authors: G. Clark Haynes, Jason Pusey, Ryan Knopf, Daniel E. Koditschek

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Technology Transfer

Dynamic Mobility via Cellular Decompositions of Coordination Spaces

Short Term Innovative Research Final Report

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1 Foreword

To accomplish feats of rapid, nimble locomotion, the ground mobile robots of tomorrow must exhibit great dexterity and dynamic mobility. As their complexity increases, however, the need for principled approaches to coordinating a robot's actuators becomes apparent. We have begun to pursue a new formalism for solving such problems, making use of Algebraic Topology and classical group theory [Hatcher, 2002, Sagan, 2001] to replace traditional methods of combinatorial search and optimization with a computationally easier and conceptually more straightforward approach that identifies and exploits symmetries in a system.

In our approach, we decompose a system into cells that index symmetric configurations. For a multilegged robot negotiating its way through a rubble-strewn terrain, the robot must choose between a number of alternative ways to move its legs. We reduce this problem to changes on a much simpler space of gait timing, using a cell complex to identify all possible gaits, discretizing the space while also providing a roadmap for gait transitions. By studying these representations—and the cellular decompositions that arise—we develop novel approaches to the fundamental control problems necessary for achieving robotic mobility.

2 List of Figures and Tables

List of Figures

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3 Problem Statement

We study the use of coordination spaces, spaces defined by mutual constraints amongst multiple dimensions in a coordinated system, as a means of structure over which planning and control can be performed. We focus specifically on the application of coordination spaces to legged robot gaits, on which constraints such as leg timing provide a

sparse structure to describe families of gaits. Over the course of the Short Term Innovative Research (STIR) grant, we have developed a greater understanding of the ways in which coordination spaces can be useful for legged robots.

We have principally achieved results along the following veins:

- Developed principled methods for enumerating cell complexes for non-trivial systems, utilizing aspects of algebraic topology, while seeking to understand the combinatorial complexity of our cellular methods for coordination spaces.
- Pursued cell complexes derived from aspects of locomotion indicative of better stability metrics and physical constraints.
- Studied the application of cell complexes to dynamical systems undergoing 2nd order dynamics.
- Developed software tools capable of organizing this information in tractable representations.

Our approach is offered in comparison with the typical, modern approach of applying grid search techniques to planning and control problems, in which each dimension of a system is discretized into multiple cells, over which grid search algorithms are utilized. Planning algorithms utilizing gridded representations have been developed for decades, with extensions for kinodynamic systems, in which discretization in action space coordinates results in a lattice of paths in a gridded world (see [LaValle, 2006] for a summaries of these approaches). The problem statement we address in this research is in developing new methods that allow us to understand better the nature of coordination spaces, deducing novel, non-gridded representations over which planning and control may be potentially performed, with the specific application of legged robots in mind.

4 Summary of Results

We describe in detail the areas in which we have developed a greater understanding over the 9 month STIR effort.

Composition and Complexity of Gait Coordination Spaces

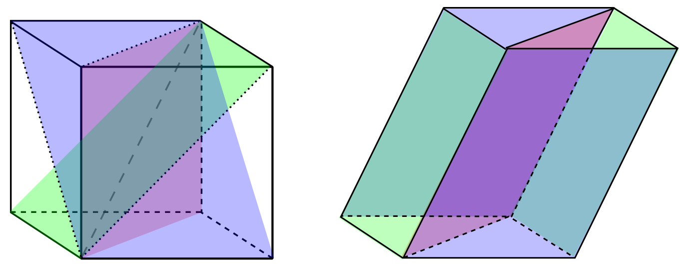
The first result we have produced focuses specifically on the ways in which classical group theory and Algebraic Topology inform the structure to our coordination spaces. We begin with a short primer on cell complexes.

Cell complexes, whether they be simplicial-, δ-, or CW-complexes, are defined by topological relationships amongst individual cells. Each cell may contain a boundary and a coboundary. The boundary is the set of bordering cells of lower-dimension, for instance, with a triangle, the triangle is bordered by 3 lines, it's boundary. A coboundary is the inverse relationship, the higher dimension cells for which a given cell is on the boundary. This cellular representation provides structure and topological blueprints for a space.

We have begun by building upon previous results denoting the "Gait Complex" for legged gaits. In this first version, described as $\mathbf{G}(\mathbb{T}^N)$ in [Haynes et al., 2009], phase equality between leg pairs provides a discretizing constraint, out of which a natural cellular decomposition arises. A simple constraint is the timing relationship between two legs, for instance two legs touching the ground at the same time in a gait. The coboundary of this constraint, it follows, are the two orderings possible, first one leg hits then the other, and the reverse. The relationships of these boundaries and coboundaries define the cell complex, and we have identified these relationships for additional types of constraints possible with legged robots, as described in the next section. As part of our efforts to highlight the use of the "Gait Complex" and coordination spaces in general, the paper [Haynes et al., 2012c] (preprint included in this report) has been accepted for publication in the International Journal of Robotics Research, the first archival presentation of these ideas of control systems built around coordination spaces on gaits. Fig. 1 shows an examples of deriving versions of the gait complex, $\mathbf{G}(\mathbb{T}^N)$.

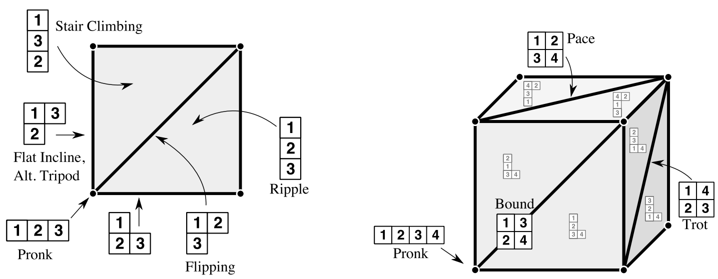
Analytical computation of the boundary and coboundary of cells is a critical problem in the development of cell complexes. In the software tools section, we describe our progress in this realm. Furthermore, we have shown that, when attempting to "join" two spaces together, via a Cartesian product, it is possible to do so in a straightforward way by performing the Cartesian product on the cell complexes for each space, thus resulting in a third of greater complexity. This is additionally described in our next section.

Along with these neighboring relationships in the cell complex descriptions of coordination spaces, we have spent efforts studying the overall complexity of these systems. Unlike purely discretized systems in which each dimension is



(a) A three-dimensional torus with leg equalities noted

(b) Same torus in an affine coordinate system



(c) The Gait Complex for the three legged system

(d) The Gait Complex for a quadrupedal system

Figure 1: Derivation of the Gait Complex for $\mathbb{T}^3$, along with the Gait Complex for $\mathbb{T}^4$.

separated into a specific number of "bins", coordination space methods are defined by the interrelationships amongst axes. Rather than geometric growth of gridded representations, in which each dimension adds a fixed multiplier in the overall cardinality in the number of cells, we have now been able to show that coordination spaces result in faster, combinatorial growth, due to the "factorial"-like nature of these interrelationships. Consider for example, the gaits with duty factor cell complex that we have studied, $\mathbf{G}(\mathbb{T}^{2N} \times \mathbb{I}^N)$, compared with our original gait complex. As shown in Table 1, the rate at which these systems grow in higher dimensions is startling.

This has brought up our first serious challenge with addressing the use of coordination spaces for planning and control. They provide very simple structure in low dimensions, but, for higher dimensions, may be providing too much structure to be of much use. We address future possibility for means of addressing this in our future work.

In addition to this disadvantage of combinatorial complexity, we have further found additional negatives to the approach of coordination spaces, namely that one cannot always blindly perform Cartesian products of two spaces to result in a third. For instance, in the gaits with duty factor cell complex, it is possible for the gait phase cell to be incompatible with the duty factor cell, even though these configurations may be on the boundary of perfectly

Table 1: Combinatorial Complexity of the Gait Complex

N	$\lvert\mathbf{G}(\mathbb{T}^N)\rvert$	$> \lvert\mathbf{G}(\mathbb{T}^{2N} \times \mathbb{I}^N)\rvert$
1	1	2
2	2	78
3	6	14,066
4	26	7,093,949
5	150	7,668,416,402
6	1,082	15,197,576,678,718
7	9,366	...
8	94,586	...

satisfactory cells. We have discovered this to force a laborious process of analytically pruning cells, as we shall describe in our description of the software tools developed.

Coordination Spaces for Duty Factor and Other Constraints

We begin by describing the discoveries we have made regarding the structure of the Gait Complex when including multiple aspects of gait timing, rather than just one phase per leg. In the new version of the Gait Complex, we represent legs by two phases, one the phase at which stance begins, another the phase at which flight begins (thus marking the two separate phases critical to define “legged” locomotion), while also including duty factor in our tableaux. We represent these gait cells as individual Young Tableaux, as used in [Haynes et al., 2009] and originally invented for the study of the symmetric group [Sagan, 2001]. We now describe these specific tablaeux as follows. Let a Cartesian product of two tableux be defined for a two legged system. The first tableau defines the phase relationships of the legs, where a is the phase at which the first leg begins stance, α when it ends stance, b the second leg begins, and β the second leg ends. For the second tableau, a and b refer to the duty factors of each leg. Items in the same row in a tableau are equal in value, while the ordering of rows defines an ordering relationship amongst groups of items. An example tableau is as follows:

$$C = \left\{ \begin{array}{|c|c|} \hline b & \alpha \\ \hline a & \beta \\ \hline \end{array}, \begin{array}{|c|c|} \hline b & a \\ \hline \end{array} \right\} \tag{1}$$

In this case, this describes both the phasing and duty factor of a perfect 50% walking gait, in which the legs transfer support at exactly the same time. Furthermore, as the duty factor values (the second tableau) have the same value, it must be a 50% duty factor gait. The power of the cellular description is the ability to define boundaries and coboundaries, for instance derived in the tableaux software we have developed:

$$\begin{aligned} \delta(C) &= \emptyset \qquad (2) \\ \delta^{-1}(C) &= \left\{ \left\{ \begin{array}{|c|c} \cline{1-1} \alpha \\ \cline{1-1} b \\ \hline a & \multicolumn{1}{c|}{\beta} \\ \hline \end{array}, \begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \right\}, \left\{ \begin{array}{|c|c} \cline{1-1} \alpha \\ \hline a & \multicolumn{1}{c|}{\beta} \\ \hline b \\ \cline{1-1} \end{array}, \begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \right\}, \left\{ \begin{array}{|c|c|} \hline b & \alpha \\ \hline \beta \\ \cline{1-1} a \\ \cline{1-1} \end{array}, \begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \right\}, \left\{ \begin{array}{|c|c|} \hline b & \alpha \\ \hline a \\ \cline{1-1} \beta \\ \cline{1-1} \end{array}, \begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \right\}, \left\{ \begin{array}{|c|c|} \hline b & \alpha \\ \hline a & \beta \\ \hline \end{array}, \begin{array}{|c|} \hline a \\ \hline b \\ \hline \end{array} \right\}, \left\{ \begin{array}{|c|c|} \hline b & \alpha \\ \hline a & \beta \\ \hline \end{array}, \begin{array}{|c|} \hline b \\ \hline a \\ \hline \end{array} \right\} \right\} \qquad (3) \end{aligned}$$

In this scenario, the walking gait has no boundary, as there is no simpler (lower dimensional) neighboring gait. It does, however, have a variety of gaits that are higher dimension and that lie on the coboundary of the walking gait. These are the various permutations of ways in which rows of the tableaux can be split, and are defined for both changing the phasing of the legs as well as changing the duty factor.

A greater level of understanding for these systems, however, has come about by studying in depth gait tableaux such as these. For instance, it is not always possible for the boundaries on the individual tableau to translate to boundaries on the composed tableau. In the case of phasing and duty factor, it is sometimes possible for the phasing to require that duty factor a is greater than duty factor b, whereas the duty factor tableau would require the opposite. For this reason, the algorithmic complexity of computing these exact sets has been elusive, but for which we have developed various techniques for pruning the tableau to only the “legal” set. With this new understanding in hand, we have started to apply these symbolic representations of more advanced gaits for as many as 4 legs (on which there are a very large number of total cells, as discussed above) and have begun analyzing the static stability of the cells, in order to compare these tableaux with prior work performed in [Haynes et al., 2009]. This work remains ongoing, however, and is currently the focus of future efforts, beyond the short term nature of the STIR effort.

We have additionally pursued describing velocity and acceleration constraints, critical elements to any actuation system, using cellular representations. In this representation, and, for now, considering only velocity constraints, each axis can be represented by one of several values: the actuator is at maximum velocity, the actuator is at 0 velocity, the actuator is at negative maximum velocity, or the actuator is on one of the two intervals defined between these values. Thus, for a single actuator, 5 cells are used to describe the possible scenarios. When two actuators are considered together, a total of 33 cells exist, 5×5 due to the Cartesian nature of the product, but with an additional 8 more cells defined by the two actuators having velocities equal to one another, as shown in Fig. 2, more unique constraints that the coordination space produces. We have used this information to inform the possibility of transitioning from one gait to another, but only on a limited, case-by-case basis, and have not yet applied these ideas to the structure of an entire complex. Along the lines of the argument of combinatorial complexity that coordination spaces produce, we do not yet consider this route to be the most fruitful to focus on quite yet, but continue to pursue it in future work.

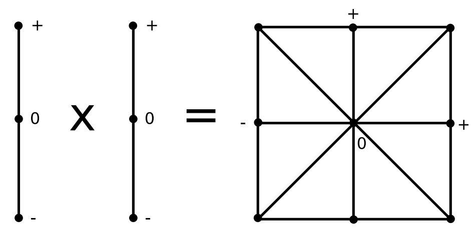


Figure 2: The Cartesian product of two velocity axes, each defined by 5 cells, results in a 33 cell complex, describing the various relationships between the two axes.

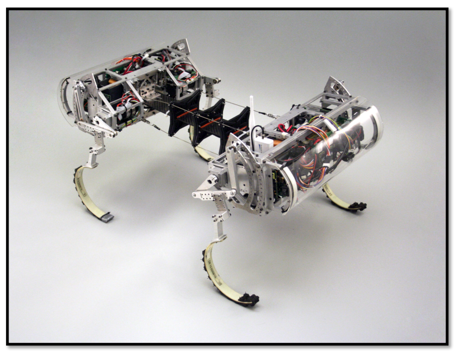


Figure 3: The Canid robot, composed of two body sections connected with a actively-driven, compliant spine.

Cellular Descriptions of Dynamical Legged Systems

Given the overall combinatorial complexity, and our current thinking that straightforward application of cell complexes directly may not result in tractable solutions, we have chosen to study the application of cell complexes for dynamical systems from a top-down, rather than bottom-up, approach. In this approach, we have developed Lagrangian simulations of dynamical systems, and have sought to use system constraints to identify the ways in which cells may exist. Useful gaits have been found to exist at the intersections of cells, while also potentially providing blueprints for developing new controllers for these behaviors. The cell complexes derived, while simplistic in nature, can be considered as potentially invariant submanifolds of the complete (and potentially intractably complex) cellular decomposition of the gait complex including many different constraints, $\mathbf{G}(A \times B \times C \times \ldots)$ (where A, B, and C represent unique dimensions included in the space).

As a simple experiment to begin with, we developed a Lagrangian simulation of the Spring Loaded Inverted Pendulum model, a simple mass sitting atop a massless leg spring that is able to touch the ground at a toe point. The low number of model degrees of freedom in the SLIP model have accurately described the vertical displacement of many biological [Blickhan and Full, 1993] and robotic [Altendorfer et al., 2000] runners. This simulation allowed us to study the ways in which a traditional Raibert controller is able to control, separately, the forward running speed and hopping height of the SLIP model, thus providing insight into the ways in which desired gaits can be considered to be "cells" within an overall cell complex, the beginning of a top-down approach to cellular decompositions we pursued next.

We next developed a much more complex simulation, Lagrangian in nature, of a back-bending planar quadruped performing dynamic locomotion. In this simulation both leg compliance as well as spine compliance play key roles in the locomotion produced. This model, based upon the Canid platform [Haynes et al., 2012a] developed through the ongoing collaboration between researchers at the University of Pennsylvania and the Army Research Laboratory (ARL), as part of the Robotics Collaborative Technology Alliance, ARL Cooperative Agreement Number W911NF-10-2-0016, has begun to describe the first such models of stable locomotion that such a robot could produce, bending its back in concert with its legs. Supported in part by the STIR effort, we have published and presented portions of this work in [Haynes et al., 2012a] and [Haynes et al., 2012b] (the latter requiring internal NREC funds to cover travel expenses beyond the $500 allowable by the STIR grant).

Our Lagrangian model, shown in Fig. 4, is of two masses, each attached to a linear spring leg, connected together

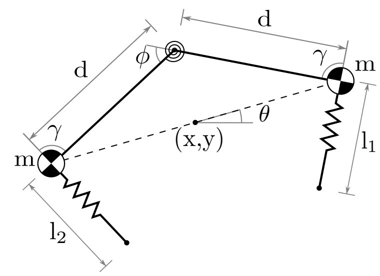


Figure 4: Lagrangian model used to derive a planar simulation of dynamic bounding.

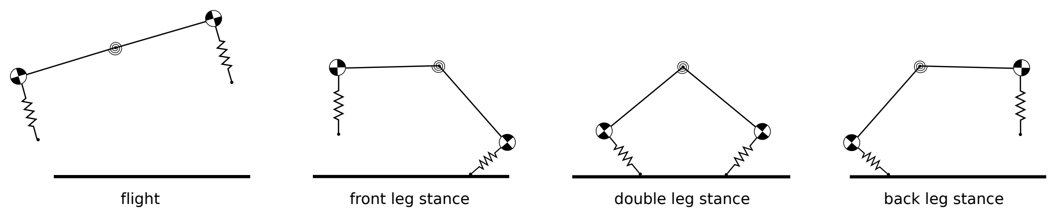


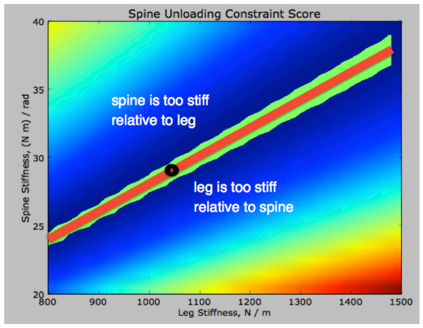
Figure 5: The four states of the model simulation

via a torsional spring in the backbone. This model includes the following states: x and y describe the horizontal and vertical displacement of the center of mass. θ is the angle between masses. l_1, l_2, and ϕ describe the positions of the two linear springs on the legs, and the torsional spring on the back. Parameters (as they do not vary with time) include: m, the mass of each of the two point masses, acceleration g due to gravity, the distance d between each mass and the center of the back, γ the rigid angle between the leg segment and each backbone segment, the rest length l_0 of each leg, the stiffness k for the legs, the rest angle ϕ_0 of the spine, and, finally, the torsional stiffness k_t of the spine. These 6 states and 8 parameters are used to define the non-dissipative equations of motion via the Lagrangian method. Two terms of kinetic energy arise from the motions of the two masses, while potential energy consists of the gravitation potential of the two masses, plus the three spring potentials when the body and legs are flexed.

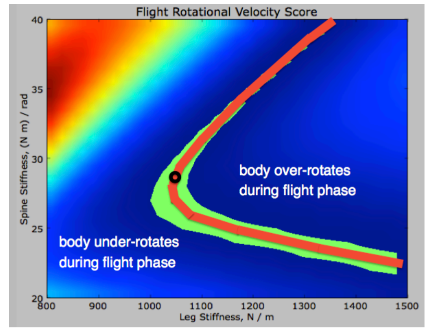
We describe four unique stance sequences in this simulation, shown in Fig. 5, including:

- Flight: consisting of 3 degrees of freedom, x, y, and θ, in which the body undergoes a ballistic trajectory. Springs are assumed to be unloaded, thus, out of 6 states, only 3 are degrees of freedom.
- Front Leg Stance: out of the 6 states, only one is not utilized, the back leg spring, as it is not loaded. The front leg stance, however, involves two constraints (the toe position of the front leg being fixed), thus there are only 3 degrees of freedom. These can be chosen arbitrarily, but we found good results to use the angle of the front leg relative to the ground, $q_1 = \pi - \gamma - \phi/2 + \theta$, the leg length l_1, and the sum of angles $q_2 = q_1 + \gamma + \phi$, as the generalized coordinates before deriving the Lagrangian EOM.
- Double Leg Stance: By far the most difficult stance to compute, this state only has 2 degrees of freedom. As all 3 springs are utilized, all state values are time-varying, however, the constraints by front and back toe positions reduce the overall freedoms to 2, rather than 6. The generalized coordinates used are q_1 and q_2. This was determined to work most easily for our computational approaches, compared to a variety of other 2 coordinate representations.
- Back Leg Stance: this is simply a mirror image of front leg stance, and also consists of three freedoms, as they are equivalent.

To derive a successful gait, we studied how system constraints, as are used to define coordination spaces, can be utilized to build structure around the problem and to identify successful gaits. In the case of the passive bounding model, we desire a system that loads the front leg, transfers load through the spine to the back leg, and lifts off, while



(a) Constraint on relative stiffness of leg to spine.



(b) Constraint on rotational velocity during flight for touchdown to match takeoff.

Figure 6: Numerical simulations across a wide variety of leg and spine stiffnesses, showing the existence of two constraints on the passive gait. The vertex at which the constraints meet offers a passive dynamic bounding solution, while the cells corresponding to the constraints provide a blueprint for control.

exhibiting limit cycle behavior, such that the same behavior is repeated again and again. Toward building a "cellular decomposition" of behavior, we identified two unique constraints that define successful locomotion:

- Foremost, to enter flight, in which we assume all springs are unloaded, we must successfully unload the back leg at the exact instant the spine is unloaded, $l_2 = l_0$ when $\phi = \phi_0$.
- Second, we want to have the model land on the ground at the same relative leg angle as occurs at lift-off. Derived by equating the equations for time of ballistic flight with a relation on the change in body angle during flight (based upon rotational velocity, $\dot{\theta}$), we derive a second constraint: $\dot{\theta} = \frac{-g\theta}{\sin(\arctan\frac{\dot{y}}{\dot{x}})\sqrt{\dot{x}^2+\dot{y}^2}}$.

These constraints define "cells", regions separated by lines on which each constraint is met, as shown in Figs. 6a-6b. The cells on the coboundary of each line relate whether the leg spring is too stiff or soft for the spine, as well as whether the body over- or under-rotates during flight. When composed together, however, the intersection of the two lines has a boundary element, a single vertex at which both constraints are met. In the case of the simulations, this vertex corresponds to successful passive locomotion. This additionally suggests that the overall structure of the cell complex is potentially useful to controlling to specific, desired behaviors, and we have started to develop new control laws that take note of the robot's position within the cell complex to control the system to the desired gait. As of yet, our studies have focused on the existing passive system, but we currently plan to extend this research to study the active locomotion of the true, dissipative robot, operating in the physical world, on which we can test the control systems proposed by these cellular representations of constraints.

This simulation produces stable, passive bounding that accurately describes the motion expected from the Canid robot being developed at the Army Research Laboratory. Following from our previous arguments regarding the overall combinatorial complexity at developing high-dimensional coordination spaces, we have chosen this top-down approach, building the simulation upon which cells are defined, as a means of achieving publishable results with this model. We are still committed, however, to discovering ways in which the bottom-up approach of identifying more primitive cells and composing them together to describe overall behavior may be useful, but believe this to be beyond the scope of what is capable in the short timeframe of the STIR effort. We discuss potential future approaches in the following section, after describing our software systems.

Software Analysis Tools

Two important software packages have been developed over the course of the grant to perform the research described herein. `motis` is a set of tools to assist in the analysis of Lagrangian simulations, utilized to develop the Canid simulation. In this package, systems are described in terms of their masses, linkages, and energies, out of which the

equations of motion are derived and compiled for extremely fast execution. This software allowed for rapid prototyping of simulations, as well as for numerical tests that provided the structure with regard to simulation constraints defining a cell complex.

A second piece of software has been written for analyzing gait tableaux. This software is capable of taking the constraint-based definition of tableaux, and deducing both the boundary and coboundary cells from a given initial cell, while building the structure of the entire cell complex. Additional work has been performed to assure that the cells produced are "legal" in terms of the composition of individual tableau. Work is still underway to generalize our methods of path planning over cells to this package, however, we have successfully written the software to allow facile composition of cell complexes, as defined by the Cartesian product amongst complexes, and to measure discrete distances between cells.

Both software packages lay out important frameworks for future endeavors into the use of coordination spaces for the study of dynamical systems.

Future Work and Conclusions

There are many additional ideas arising from work with coordination spaces that are possible to study. With our experience, over the very short term STIR effort, showing that cell complexes, built from the ground up, of dynamical systems are very complex systems, with complexity growing at extremely fast rates as more dimensions are added, we have been emboldened to pursue tangential routes for proving the utility of coordination spaces. We believe now that is it not easily tractable to understand an entire cell complex a priori, and, instead, it should be computed on the fly, as is performed in our tableaux software. Along with this understanding, however, is the need to incorporate top-down approaches into the study of coordination and constraints, studying locomotion produced by systems to derive which specific axes (or manifolds) within a space are the most important. We believe there to be future possibilities to incorporate the emerging field of computational topology [Zomorodian, 2005, Edelsbrunner and Harer, 2010] to notate these specific manifolds as appropriate.

In summary, we have pursued a variety of ways of studying the existence of constraint-based cellular decompositions of legged robot behaviors. We have been able to elucidate greater structure of intricate systems involving touchdown, liftoff, and relative timing of multiple legs in a gait, through a better understanding of Cartesian composition of gait cells, via a bottom-up approach to cellular decompositions. Deriving from system constraints and dynamical systems, we have also pursued top-down approaches for system cells, and have shown that useful gaits exist at intersections of cells, and on which control systems may be developed. Finally, a set of software tools, combined with a greater understanding of the topology of these systems, suggests a variety of potential pursuits to extend this work in the future.

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